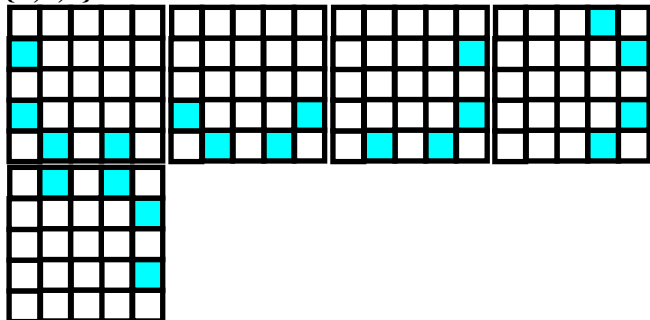


4-cycles and 2 2-cycles: Positions

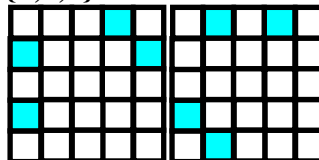
Unfortunately, we cannot make the same shortcut that we did for 3-cycles. Here we must find $\binom{8}{4} = 70$

different combinations. Of course like the 3-cycles, there will be several equivalent cases which will cancel to reduce the amount of positions significantly.

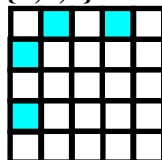
{0,0,0}



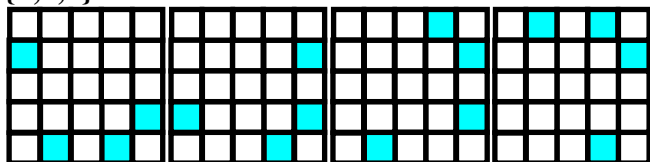
{0,3,0}



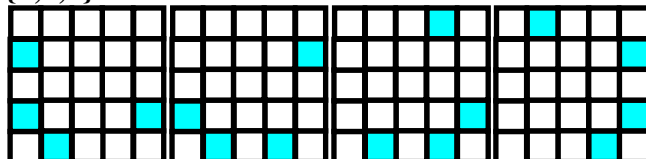
{0,4,0}



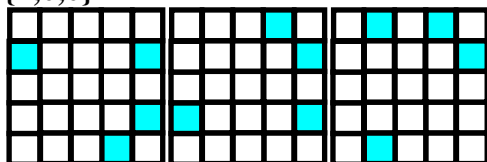
{1,0,0}



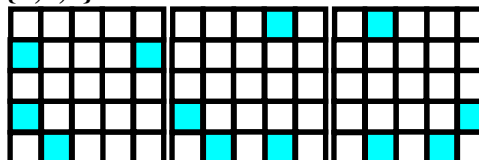
{0,0,1}



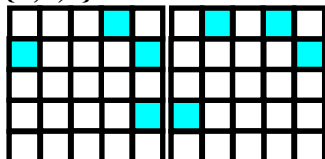
{2,0,0}



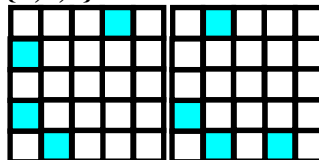
{0,0,2}



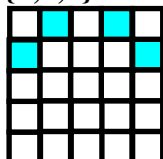
{3,0,0}



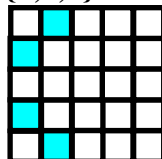
{0,0,3}



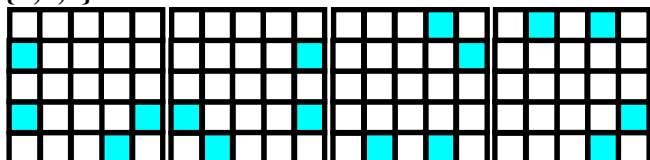
{4,0,0}



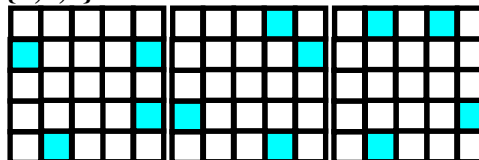
{0,0,4}



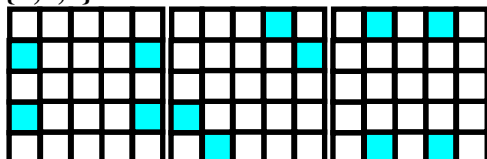
{0,1,0}



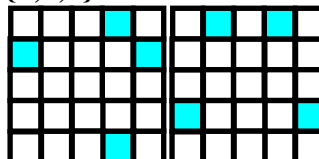
{1,1,0}



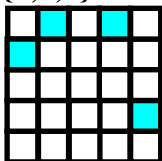
{0,2,0}



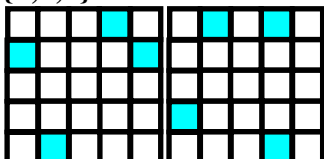
{2,1,0}



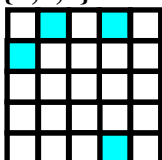
{3,1,0}



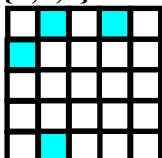
{1,2,0}



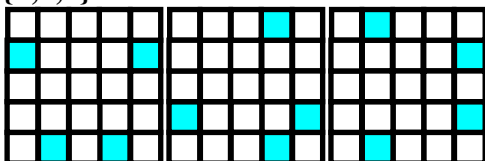
{2,2,0}



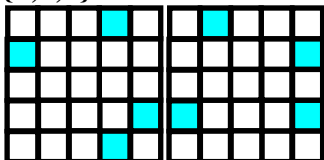
{1,3,0}



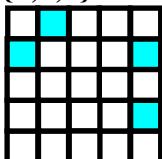
{1,0,1}



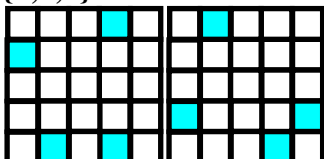
{2,0,1}



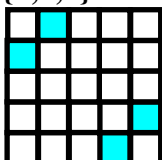
{3,0,1}



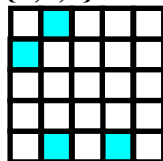
{1,0,2}



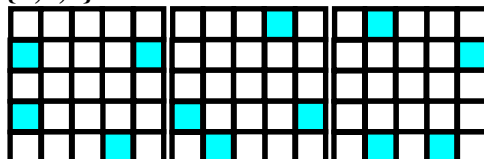
{2,0,2}



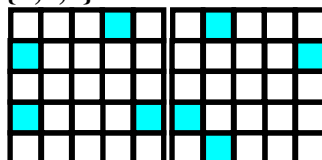
{1,0,3}



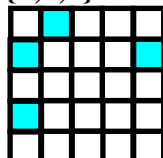
{0,1,1}



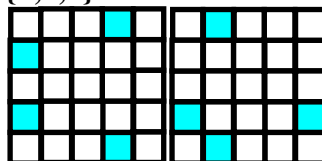
{0,2,1}



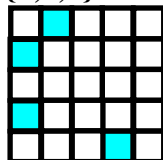
{0,3,1}



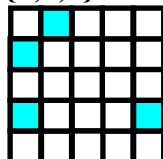
{0,1,2}



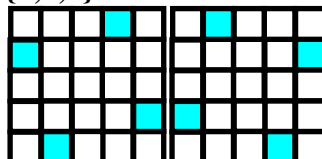
{0,1,3}



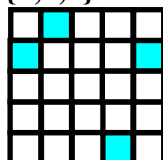
{0,2,2}



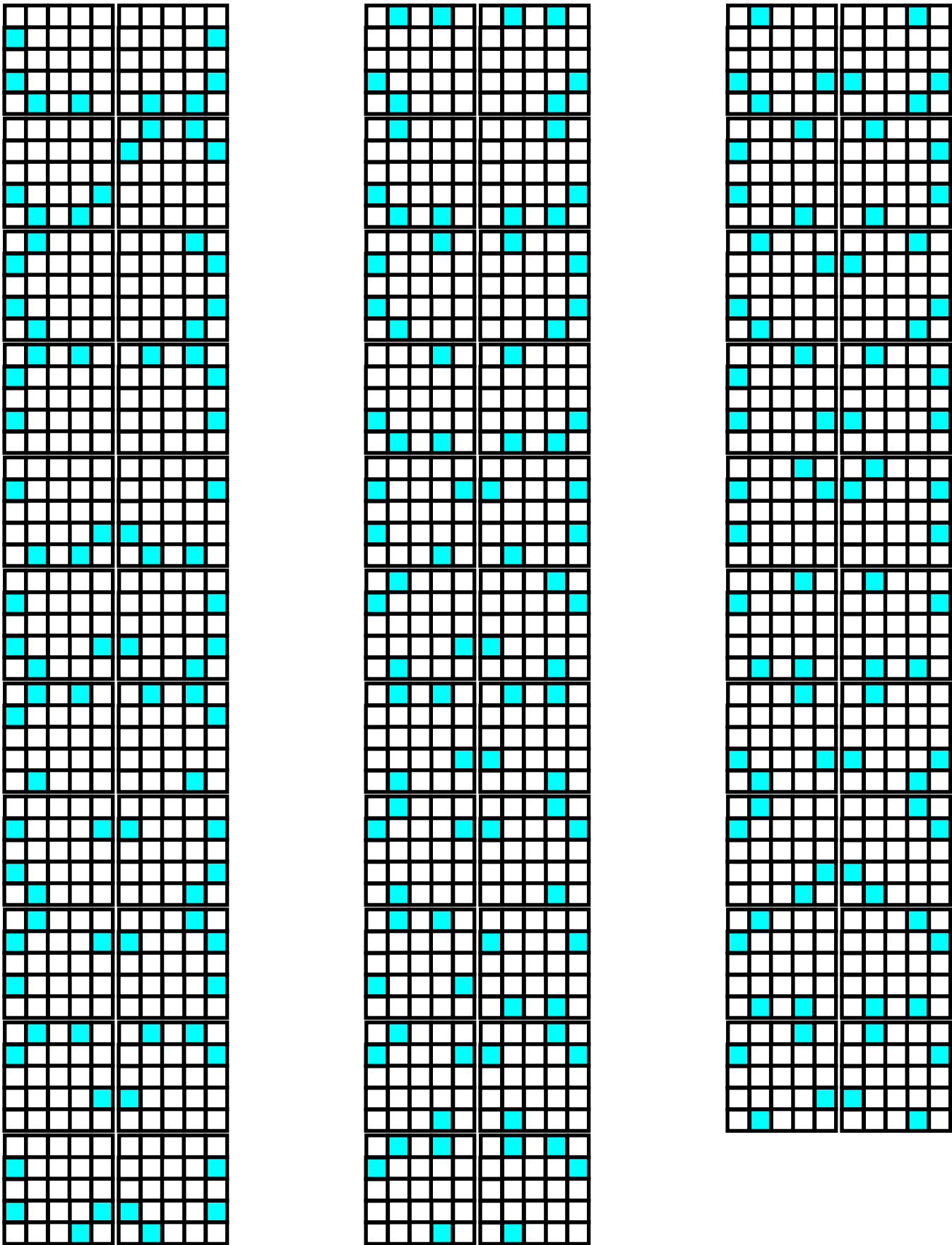
{1,1,1}

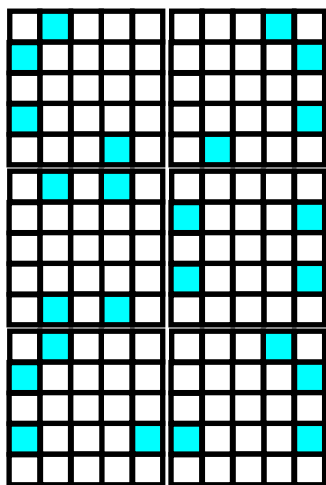


{2,1,1}



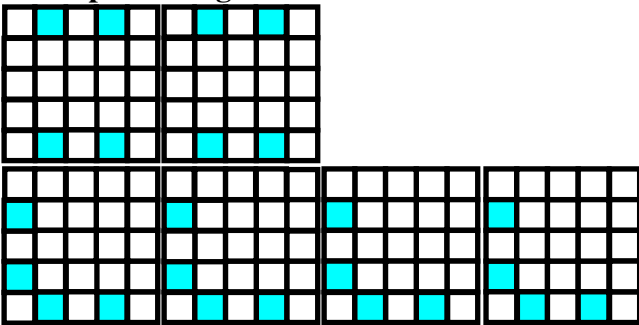
Group with mirrors



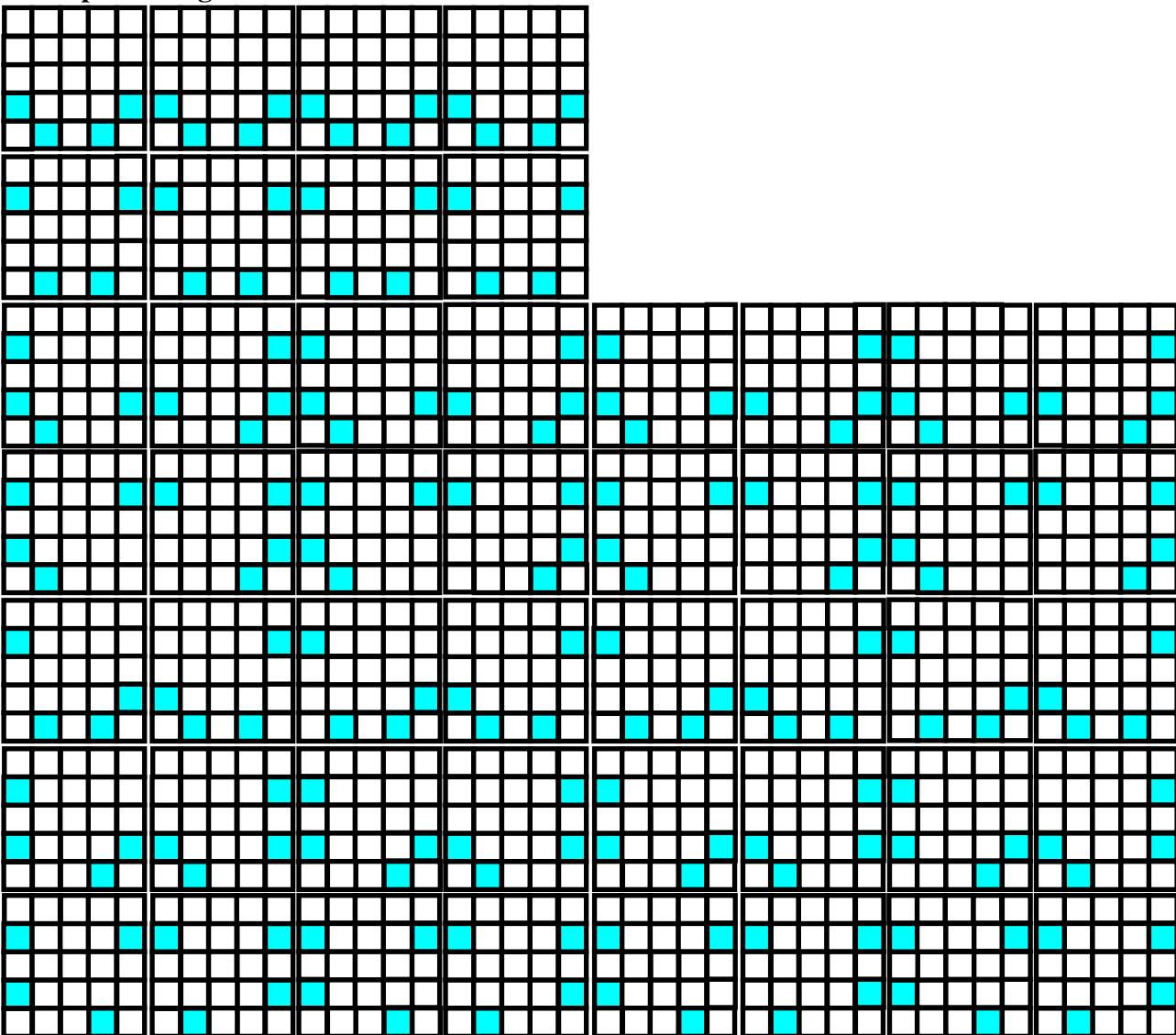


Rotating to match up repeated ones: (as a matter of fact, I rotated all so that, if they can be contained in the bottom 3 composite edges, they were...so that I could see equivalences better in the next step). Also, I grouped together the cases with two composite edges, 3 composite edges, and 4 composite edges, to even further see all cancellations easier.

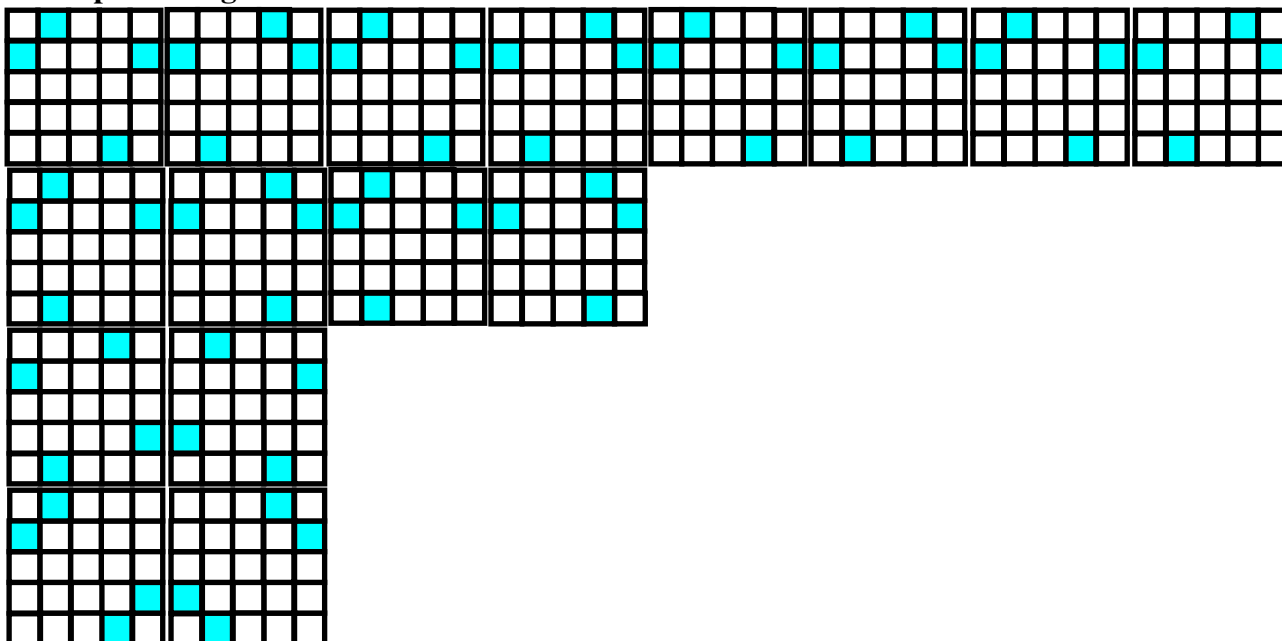
2 Composite Edges



3 Composite Edges



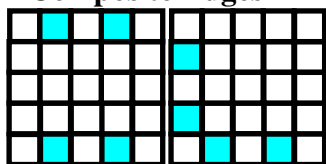
4 Composite Edges



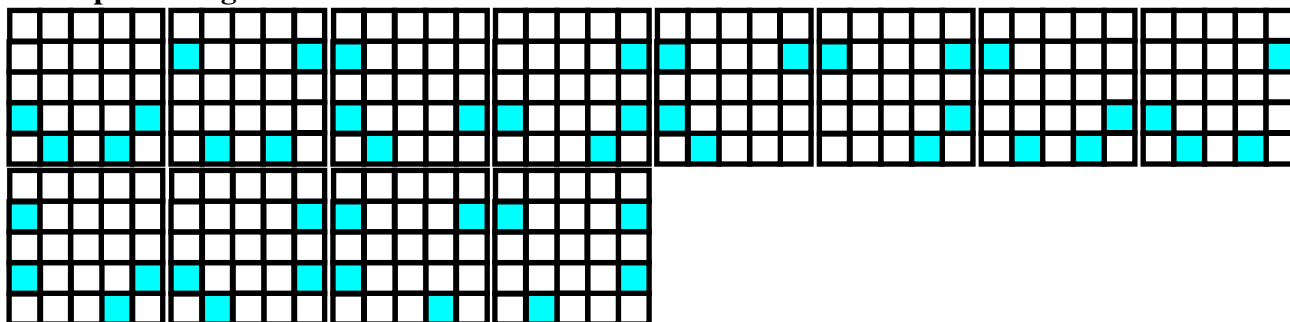
Deleting Equivalences, we go from 70 $\rightarrow$ 21

$$70 - (1 + 3 + 3 + 3 + 5(6) + 6 + 2) - 1 = 21$$

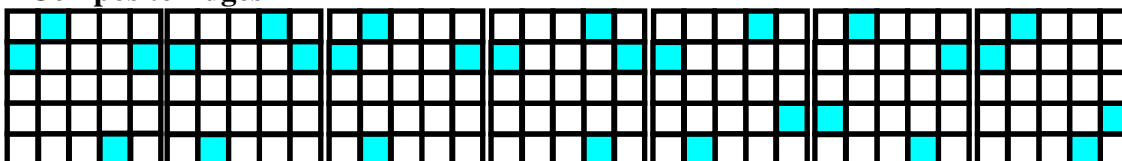
2 Composite Edges



3 Composite Edges



4 Composite Edges



Moving of the 4 pieces in those 21 distinct positions

Just as permutations were used for 3-cycles, though it turned out that there is only clockwise or counterclockwise with them, permutations are also used to find permutations of 4 pieces. That is, both 2 2-cycles and 4-cycles.

Here are the 4! Permutations for four objects

1, 2, 3, 4	2, 1, 3, 4	3, 1, 2, 4	4, 1, 2, 3
	2 2-cycle	4-cycle	
1, 2, 4, 3	2, 1, 4, 3	3, 1, 4, 2	4, 1, 3, 2
1, 3, 2, 4	2, 3, 1, 4	3, 2, 1, 4	4, 2, 1, 3
	4-cycle		
1, 3, 4, 2	2, 3, 4, 1	3, 2, 4, 1	4, 2, 3, 1
	2 2-cycle	4-cycle	
1, 4, 2, 3	2, 4, 1, 3	3, 4, 1, 2	4, 3, 1, 2
	4-cycle		
1, 4, 3, 2	2, 4, 3, 1	3, 4, 2, 1	4, 3, 2, 1
	4-cycle	2 2-cycle	

Overview

- As we have seen, there are a total of 21 distinct positions of 4 pieces in the last layer of a big cube.
- Each of those distinct positions has 9 possible permutation cases (three 2 2-cycles and six 4-cycles).

Even Permutations (2 2-cycles)

- {3, 4, 1, 2} (Commonly known as “even/permutation parity”)
- {4, 3, 2, 1} (Commonly known as “even/permutation parity with orientation”)
- {2, 1, 4, 3} (Commonly known as “two edge orientation”)

Odd Permutations (4-cycles)

- {4, 1, 2, 3} (All 4 pieces need to be permuted in a clockwise fashion)
- {2, 3, 4, 1} (All 4 pieces need to be permuted in a counterclockwise fashion)
- {3, 4, 2, 1} (“Double Parity 1”)
- {4, 3, 1, 2} (“Double Parity 2”)
- {2, 4, 1, 3} (“Weird Case 1”)
- {3, 1, 4, 2} (“Weird Case 2”)

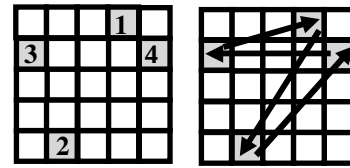
- Thus, there are a maximum of $(21)(9) = 189$ cases between 2 2-cycles and 4-cycles. However, after some cancellations, it turns out that there are a total of **58** 2 2-cycle cases and **110** 4-cycle cases. Again, these are distinct cases restricted just to the last layer.
- The following is how I treat these 9 different permutation cases for all 21 positions previously derived.
- Given that the solved state is a sequence {1, 2, 3, 4} from the left slot in the front composite edge counterclockwise to the bottom slot on the left composite edge, each and every case has the following sequences which need to be permuted back so that they have the following sequence:

{1, 2, 3, 4}:

- For example, the figure to the right represents the solved state of the 4 wing edge pieces in this particular case. (I did it in a counterclockwise fashion, but doing it clockwise will still produce the same results, even if in a different order).
- Now I feel it is important to mention how I specified the direction of the arrows in the figures for each permutation. I will give an example to illustrate. Suppose I want to find how the arrows go to the position shown to the right above for the permutation {2, 4, 1, 3}. The first step is to see what the goal is. The goal for all figures is to make the particular permutation go to {1, 2, 3, 4}. In order to do this for the permutation {2, 4, 1, 3}, there needs to be:

			3	
4				2
	1			

an arrow starting from location 2 and ending at location 4;
 an arrow starting from location 4 and ending at location 3;
 an arrow starting from location 3 and ending at location 1; and
 an arrow starting from location 1 and ending at location 2.



Or simply: $2 \rightarrow 4$, $4 \rightarrow 3$, $3 \rightarrow 1$, and $1 \rightarrow 2$.

For every table, the even permutations (2 2-cycles) are given first, followed by the odd permutations (4-cycles).

Scenario 1

2 2-cycles	
Permutations	Positions
{3, 4, 1, 2} (Commonly known as “even/permutation parity”): $3 \leftrightarrow 1$, $4 \leftrightarrow 2$	
{4, 3, 2, 1} (Commonly known as “even/permutation parity with orientation”): $4 \leftrightarrow 1$, $3 \leftrightarrow 2$	
{2, 1, 4, 3} (Commonly known as “two edge orientation”): $2 \leftrightarrow 1$, $4 \leftrightarrow 3$	

4-Cycles

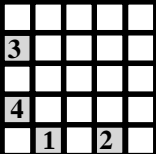
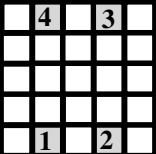
Permutations	Positions
{4, 1, 2, 3} (All 4 pieces need to be permuted in a clockwise fashion): $4 \rightarrow 3$, $3 \rightarrow 2$, $2 \rightarrow 1$, $1 \rightarrow 4$	
{2, 3, 4, 1} (All 4 pieces need to be permuted in a counterclockwise fashion): $2 \rightarrow 3$, $3 \rightarrow 4$, $4 \rightarrow 1$, $1 \rightarrow 2$	
{3, 4, 2, 1} (“Double Parity 1”): $3 \rightarrow 2$, $2 \rightarrow 4$, $4 \rightarrow 1$, $1 \rightarrow 3$	
{4, 3, 1, 2} (“Double Parity 2”): $4 \rightarrow 2$, $2 \rightarrow 3$, $3 \rightarrow 1$, $1 \rightarrow 4$	
{2, 4, 1, 3} (“Weird Case 1”): $2 \rightarrow 4$, $4 \rightarrow 3$, $3 \rightarrow 1$, $1 \rightarrow 2$	
{3, 1, 4, 2} (“Weird Case 2”) : $3 \rightarrow 4$, $4 \rightarrow 2$, $2 \rightarrow 1$, $1 \rightarrow 3$	

Scenario 2: Contains the mirrors of Scenario 1 but the figures still match the permutations previously shown.

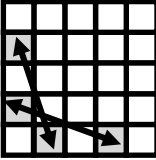
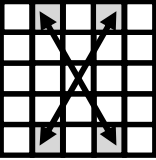
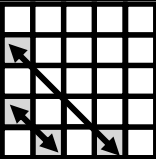
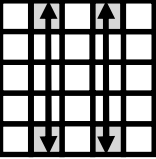
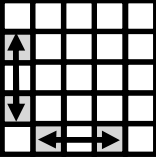
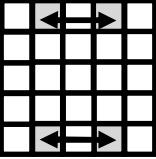
Note: Here is a tip that can help you to match the 4-cycle figures with their corresponding permutations easier. Besides the first two permutations which are clockwise and counterclockwise respectively, the remaining four can be identified with only one piece of information. It is indicated in bold next to each permutation below:

{3, 4, 2, 1} (“Double Parity 1”): **4→1**
 {4, 3, 1, 2} (“Double Parity 2”): **2→3**
 {2, 4, 1, 3} (“Weird Case 1”): **1→2**
 {3, 1, 4, 2} (“Weird Case 2”): **3→4**

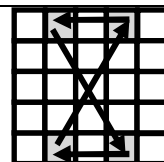
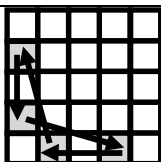
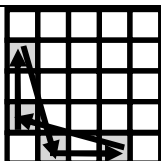
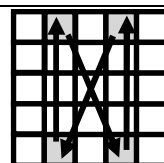
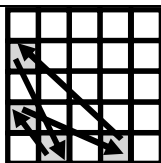
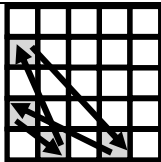
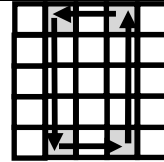
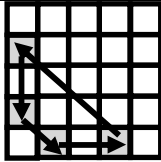
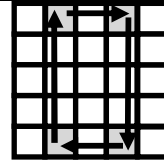
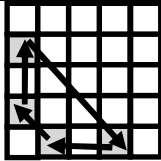
In 2 Composite Edges: 16 Cases

					
--	---	--	--	---	--

Even Permutations (2 2-Cycles): 6 Cases

Odd Permutations (4-Cycles): 10 Cases



In 3 Composite Edges: 108 Cases

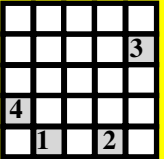
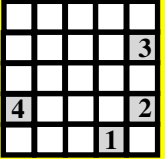
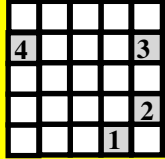
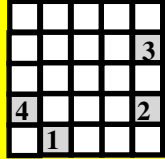
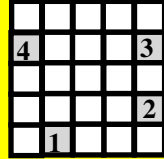
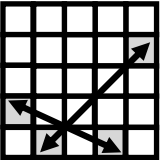
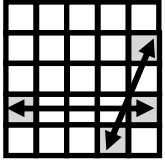
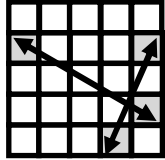
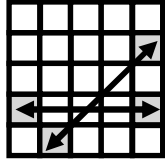
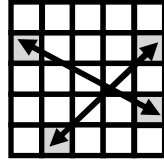
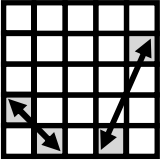
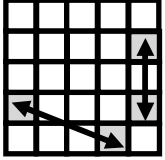
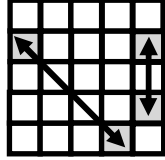
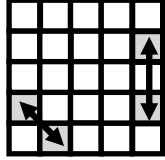
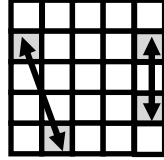
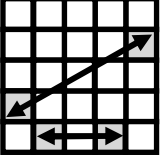
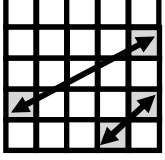
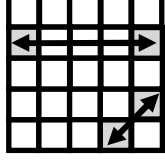
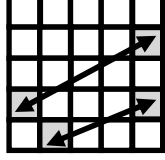
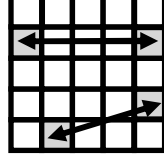
Scenario 1: 63 Cases

--	--	--	--	--	--	--

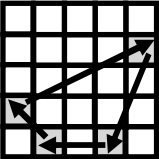
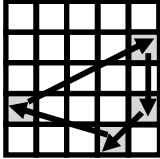
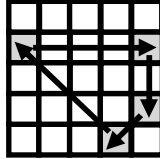
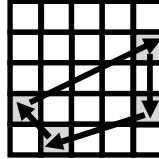
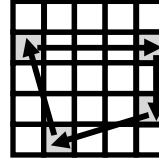
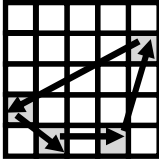
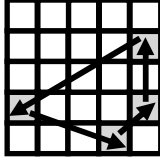
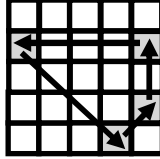
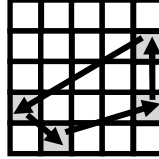
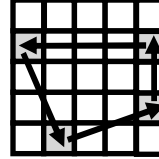
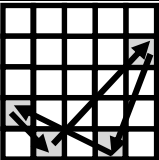
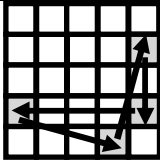
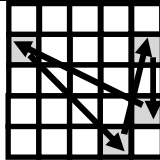
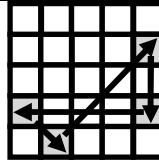
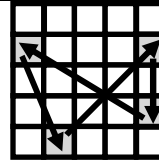
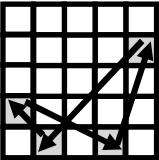
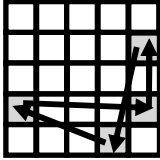
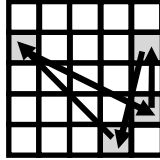
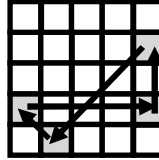
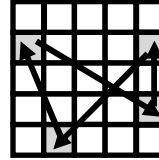
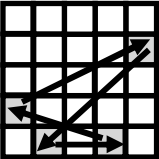
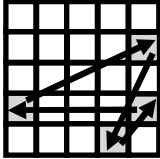
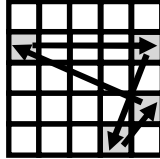
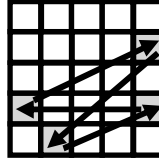
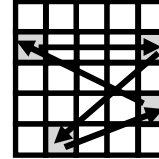
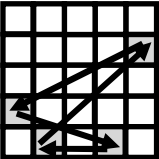
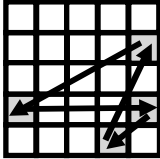
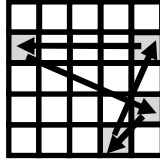
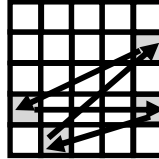
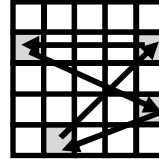
Even Permutations (2 2-Cycles): 21 Cases

Odd Permutations (4-Cycles): 42 Cases

Scenario 2: Mirror of Scenario 1: 45 Cases

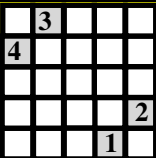
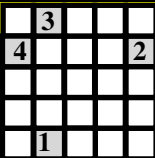
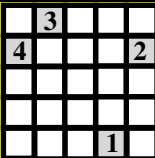
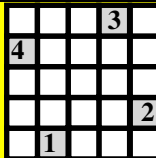
						
Even Permutations (2 2-Cycles): 15 Cases						
						
						
						

Odd Permutations (4-Cycles): 30 Cases

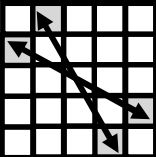
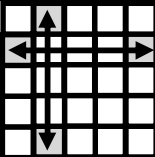
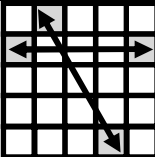
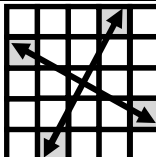
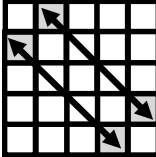
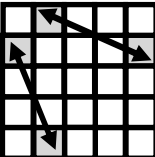
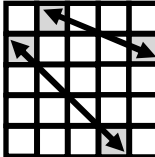
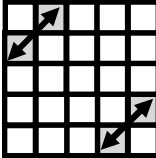
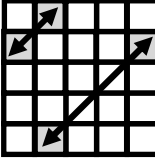
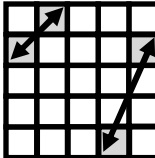
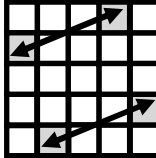
						
						
						
						
						
						

In 4 Composite Edges: 44 Cases

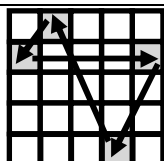
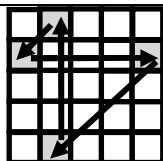
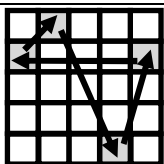
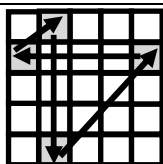
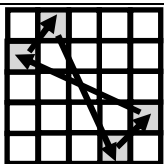
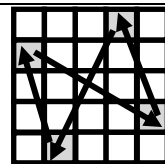
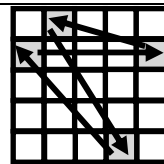
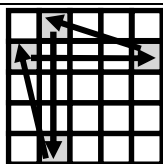
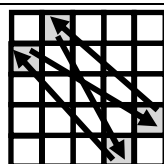
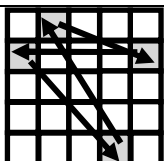
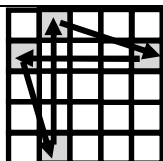
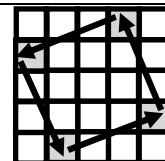
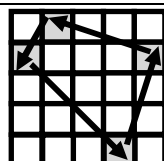
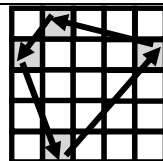
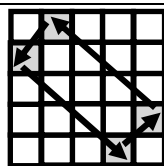
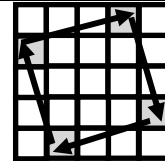
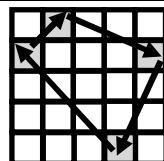
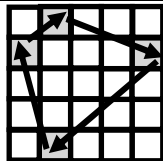
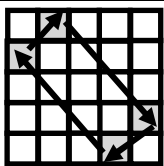
Scenario 1: 30 Cases

							
--	--	---	--	--	--	---	--

Even Permutations (2 2-Cycles): 11 Cases

Odd Permutations (4-Cycles): 19 Cases



Scenario 2: Mirror of Scenario 1: 14 Cases

Even Permutations (2 2-Cycles): 5 Cases						

Odd Permutations (4-Cycles): 9 Cases

